1. P represents the variable complex number z. Find the locus of P, if

(i) $Im\left[\frac{2z+1}{iz+1}\right] = -2$ (ii) $|z-5i| = |z+5i|$

(iii) $Re\left(\frac{z-1}{z+i}\right) = 1$ (iv) $|2z-3| = 2$ (v) $\arg\left(\frac{z-1}{z+3}\right) = \frac{\pi}{2}$

2 If α and β are the roots of the equation $x^2 - 2px + (p^2 + q^2) = 0$ and $\tan\theta = \frac{q}{y+p}$ show that $\frac{(y+\alpha)^n - (y+\beta)^n}{\alpha - \beta} = q^{n-1}\frac{\sin n\theta}{\sin^n\theta}$

3. . If α and β are the roots of $x^2 - 2x + 4 = 0$

Prove that $\alpha^n - \beta^n = i2^{n+1}\sin\frac{n\pi}{3}$ and deduct $\alpha^9 - \beta^9$

4. If $x + \frac{1}{x} = 2\cos\theta$ and $y + \frac{1}{y} = 2\cos\phi$ show that

(i) $\frac{x^m}{y^n} + \frac{y^n}{x^m} = 2\cos(m\theta - n\phi)$ (ii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\theta - n\phi)$

5. If $a = \cos 2\alpha + i\sin 2\alpha$, $b = \cos 2\beta + i\sin 2\beta$ and $c = \cos 2\gamma + i\sin 2\gamma$ prove that

(i) $\sqrt{abc} + \frac{1}{\sqrt{abc}} = 2\cos(\alpha + \beta + \gamma)$

(ii) $\frac{a^2\, b^2 + c^2}{abc} = 2\cos 2(\alpha + \beta - \gamma)$

6. Find all the values of the following :

(iii) $(-\sqrt{3} - i)^{\frac{2}{3}}$

Solve :

7. (ii) $x^4 - x^3 + x^2 - x + 1 = 0$

8. Find all the values of $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$ and hence prove that the product of the values is 1.

9. P represents the variable complex number z, find the locus of P if

(i) $Re\left(\frac{z+1}{z+i}\right) = 1$ (ii) $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$

10. *Example 3.22 :* If α and β are the roots of $x^2 - 2x + 2 = 0$ and $\cot\theta = y + 1$,

show that $\frac{(y+\alpha)^n - (y+\beta)^n}{\alpha - \beta} = \frac{\sin n\theta}{\sin^n\theta}$

11. *Example 3.23* : Solve the equation $x^9 + x^5 - x^4 - 1 = 0$

12. *Example 3.24* : Solve the equation $x^7 + x^4 + x^3 + 1 = 0$

13. *Example 3.25* : Find all the values of $(\sqrt{3} + i)^{\frac{2}{3}}$