INTEGRAL CALCULUS – APPLICATIONS
PART – A

QUES.NO.,	QUESTION	ANSWER
1	The Value of $\int_0^{\pi/2} \frac{\cos^{5/3}x}{\cos^{5/3}x + \sin^{5/3}x}\, dx$ is	$\frac{\pi}{4}$
2	The Value of $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x}\, dx$ is	**0**
3	The Value of $\int_0^{1} x(1-x)^4\, dx$ is	$\frac{1}{30}$
4	The Value of $\int_{-\pi/2}^{\pi/2} \left(\frac{\sin x}{2 + \cos x}\right) dx$ is	**0**
5	The Value of $\int_0^{\pi} \sin^4 x\, dx$ is	$3\pi/8$
6	The Value of $\int_0^{\pi/4} \cos^3 2x\, dx$ is	$\frac{1}{3}$
7	The Value of $\int_0^{\pi} \sin^2 x \cos^3 x\, dx$ is	**0**
8	The area bounded by the line $y = x$, the x-axis, the ordinates $x = 1$, $x = 2$ is	$\frac{3}{2}$
9	The area of the region bounded by the graph of $y = \sin x$ and $y = \cos x$ between $x = 0$ and $x = \frac{\pi}{4}$ is	$\sqrt{2} - 1$
10	The area between the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and its auxillary circle is	$\pi a(a - b)$
11	The area bounded by the parabola $y^2 = x$ and its latus rectum is	$\frac{1}{6}$
12	The volume of the solid obtained by revolving $\frac{x^2}{9} + \frac{y^2}{16} = 1$ about the minor axis is	64π
13	The volume, when the curve $y = \sqrt{3 + x^2}$ from $x = 0$ to $x = 4$ is rotated about x-axis is	$\frac{100}{3}\pi$
14	The volume generated when the region bounded by $y = x$, $y = 1$, $x = 0$ is rotated about y-axis is	$\frac{\pi}{3}$

15	Volume of solid obtained by revolving the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about major and minor axes are in the ratio	$b : a$
16	The volume generated by rotating the triangle with vertices at (0, 0), (3, 0) and (3, 3) about x-axis is	9π
17	The length of the arc of the curve $x^{2/3}+y^{2/3}=4$ is	48
18	The surface area of the solid of revolution of the region bounded by $y = 2x$, $x = 0$ and $x = 2$ about x-axis is	$8\sqrt{5}\,\pi$
19	The curved surface area of a sphere of radius 5, intercepted between two parallel planes of distance 2 and 4 from the centre is	20π

PART-B

1. **Evaluate** $\int_0^1 \log\left(\frac{1}{x}-1\right) dx$

2. **Evaluate** $\int_0^3 \frac{\sqrt{x}\,dx}{\sqrt{x}+\sqrt{3-x}}$

3. **Evaluate** $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

4. Evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$

5. Evaluate $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+f(\cos x)} dx$

6. Evaluate $\int_0^{\pi/2} \log(\tan x) dx$

PART –C

1. Derive the formula for the volume of a right circular cone with radius 'r' and height 'h'.

2. Find the length of the curve $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{a}\right)^{2/3}=1$

3.

4. Find the length of the curve $x = a(t - \sin t)$, $y = a(1 - \cos t)$ between $t = 0$ and π.

5. Find the surface area of the solid generated by revolving the arc of the parabola $y^2 = 4ax$, bounded by its latus rectum about x-axis.

6. Prove that the curved surface area of a sphere of radius r intercepted between two parallel planes at a distance a and b from the centre of the sphere is $2\pi r\ (b - a)$ and hence deduct the surface area of the sphere. $(b > a)$.

7. ***Example 7.40:*** Find the surface area of the solid generated by revolving the cycloid $x = a(t + \sin t)$, $y = a(1 + \cos t)$ about its base (x-axis).

8. ***Example 7.39:*** Show that the surface area of the solid obtained by revolving the arc of the curve $y = \sin x$ from $x = 0$ to $x = \pi$ about x-axis is $2\pi\left[\sqrt{2} + \log\left(1 + \sqrt{2}\right)\right]$

DIFFERENTIAL EQUATIONS
PART - A

QUES.NO.,	QUESTION
1.	The integrating factor of $\frac{dy}{dx} + 2\frac{y}{x} = e^{4x}$ is
2.	If cos x is an integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$ then $P =$
3.	The integrating factor of $dx + xdy = e^{-y} \sec^2 y \, dy$ is
4.	Integrating factor of $\frac{dy}{dx} + \frac{1}{x \log x} . y = \frac{2}{x^2}$ is
5.	Solution of $\frac{dx}{dy} + mx = 0$, where $m < 0$ is
6.	$y = cx - c^2$ is the general solution of the differential equation
7.	The differential equation $\left(\frac{dx}{dy}\right)^2 + 5y^{1/3} = x$ is
8.	The differential equation of all non-vertical lines in a plane is
9.	The differential equation of all circles with centre at the origin is
10.	The integrating factor of the differential equation $\frac{dy}{dx} + py = Q$ is
11.	The complementary function of $(D^2 + 1)y = e^{2x}$ is
12.	A particular integral of $(D^2 - 4D + 4)y = e^{2x}$ is
13.	The differential equation of the family of lines $y = mx$ is
14.	The degree of the differential equation $\sqrt{1 + \left(\frac{dy}{dx}\right)^{1/3}} = \frac{d^2y}{dx^2}$
15.	The degree of the differential equation $c = \frac{\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{2/3}}{\frac{d^3y}{dx^3}}$ where c is a constant is
16.	The amount present in a radio active element disintegrates at a rate proportional to its amount. The differential equation corresponding to the above statement is (k is negative)
17.	The differential equation satisfied by all the straight lines in xy plane is
18.	If $y = ke^{\lambda x}$ then its differential equation is
19.	The differential equation obtained by eliminating a and b from $y = ae^{3x} + be^{-3x}$ is
20	The differential equation formed by eliminating A and B from the relation $y = e^x (A \cos x + B \sin x)$ is

21	If $\frac{dy}{dx} = \frac{x-y}{x+y}$ then
22	If $f'(x) = \sqrt{x}$ and $f(1) = 2$ then $f(x)$ is
23	On putting $y = vx$, the homogeneous differential equation $x^2dy + y(x+y)dx = 0$ becomes
24	The integrating factor of the differential equation $\frac{dy}{dx} - y\tan x = \cos x$ is
25	The P.I. of $(3D^2 + D - 14)y = 13e^{2x}$ is
26	The particular integral of the differential equation $f(D)y = e^{ax}$ where $f(D) = (D-a)\,g(D)$, $g(a) \neq 0$ is

PART-B

1. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^{3x}$ when $x = \log 2$, $y = 0$ and when $x = 0$, $y = 0$

2. Solve $(D^2 - 1)y = \cos 2x - 2\sin 2x$

3. Solve $(D^2 - 2D + 2)\ y = \sin 2x + 5$

4. Solve $(D^2 - 5D + 6)\ y = \sin 2x + 2e^{2x}$

5. Solve $(D^2 - 6D + 9)\ y = x + e^{2x}$

6. Radium disappears at a rate proportional to the amount present. If 5% of the original amount disappears in 50 years, how much will remain at the end of 100 years. [Take A_0 as the initial amount].

7. *Example 8.34 :* In a certain chemical reaction the rate of conversion of a substance at time t is proportional to the quantity of the substance still untransformed at that instant. At the end of one hour, 60 grams remain and at the end of 4 hours 21 grams. How many grams of the substance was there initially?

8. *Example 8.35 :* A bank pays interest by continuous compounding, that is by treating the interest rate as the instantaneous rate of change of principal. Suppose in an account interest accrues at 8% per year compounded continuously. Calculate the percentage increase in such an account over one year. [Take $e^{.08} \approx 1.0833$]

9. The sum of Rs. 1000 is compounded continuously, the nominal rate of interest being four percent per annum. In how many years will the amount be twice the original principal? ($\log_e 2 = 0.6931$).

10. The rate at which the population of a city increases at any time is proportional to the population at that time. If there were 1,30,000 people in the city in 1960 and 1,60,000 in 1990 what population may be anticipated in 2020. $\left[\log_e\left(\frac{16}{13}\right) = .2070\ ;\ e^{.42} = 1.52\right]$

11. A radioactive substance disintegrates at a rate proportional to its mass. When its mass is 10 mgm, the rate of disintegration is 0.051 mgm per day. How long will it take for the mass to be reduced from 10 mgm to 5 mgm. $[\log_e 2 = 0.6931]$

12. The number of bacteria in a yeast culture grows at a rate which is proportional to the number present. If the population of a colony of yeast bacteria triples in 1 hour. Show that the number of bacteria at the end of five hours will be 3^5 times of the population at initial time.

13. A cup of coffee at temperature 100°C is placed in a room whose temperature is 15°C and it cools to 60°C in 5 minutes. Find its temperature after a further interval of 5 minutes.

14. *Example 8.37 :* For a postmortem report, a doctor requires to know approximately the time of death of the deceased. He records the first temperature at 10.00 a.m. to be 93.4°F. After 2 hours he finds the temperature to be 91.4°F. If the room temperature (which is constant) is 72°F, estimate the time of death. (Assume normal temperature of a human body to be 98.6°F).

$$\left[\log_e \frac{19.4}{21.4} = -\,0.0426 \times 2.303 \text{ and } \log_e \frac{26.6}{21.4} = 0.0945 \times 2.303\right]$$

15. *Example 8.38 :* A drug is excreted in a patients urine. The urine is monitored continuously using a catheter. A patient is administered 10 mg of drug at time $t = 0$, which is excreted at a Rate of $-\,3t^{1/2}$ mg/h.

(i) What is the general equation for the amount of drug in the patient at time $t > 0$?

(ii) When will the patient be drug free?