

DIFFERENTIAL CALCULUS- APPLICATION II

PART – A

QUES.NO.,	QUESTION
1	If $u = x^y$ then $\frac{\partial u}{\partial x}$ is equal to
2	If $u = \sin^{-1} \left(\frac{x^4 + y^4}{x^2 + y^2} \right)$ and $f = \sin u$ then f is a homogeneous function of degree
3	If $u = \frac{1}{\sqrt{x^2 + y^2}}$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is equal to
4	The curve $y^2(x - 2) = x^2(1 + x)$ has
5	If $x = r \cos \theta$, $y = r \sin \theta$, then $\frac{\partial r}{\partial x}$ is equal to
6	<p align="center">Identify the true statements in the following :</p> <p>(i) If a curve is symmetrical about the origin, then it is symmetrical about both axes.</p> <p>(ii) If a curve is symmetrical about both the axes, then it is symmetrical about the origin.</p> <p>(iii) A curve $f(x, y) = 0$ is symmetrical about the line $y = x$ if $f(x, y) = f(y, x)$.</p> <p>(iv) For the curve $f(x, y) = 0$, if $f(x, y) = f(-y, -x)$, then it is symmetrical about the origin.</p>
7	If $u = \log \left(\frac{x^2 + y^2}{xy} \right)$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is
8	The percentage error in the 11th root of the number 28 is approximately _____ times the percentage error in 28.

9	The curve $a^2y^2 = x^2(a^2 - x^2)$ has
10	An asymptote to the curve $y^2(a + 2x) = x^2(3a - x)$ is
11	If $u = y \sin x$, then $\frac{\partial^2 u}{\partial x \partial y}$ is equal to
12	In which region the curve $y^2(a + x) = x^2(3a - x)$ does not lie?
13	If $u = f\left(\frac{y}{x}\right)$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is equal to
14	The curve $9y^2 = x^2(4 - x^2)$ is symmetrical about
15	The curve $ay^2 = x^2(3a - x)$ cuts the y -axis at

PART - B

- 1) Use differentials to find an approximate value for the given number

$$\sqrt{36.1}$$

- 2)

Use differentials to find an approximate value for $\sqrt[3]{65}$.

$$\frac{1}{10.1}$$

- 3) *Example 6.5* : The time of swing T of a pendulum is given by $T = k\sqrt{l}$ where k is a constant. Determine the percentage error in the time of swing if the length of the pendulum l changes from 32.1 cm to 32.0 cm.

- 4)

If $w = x + 2y + z^2$ and $x = \cos t$; $y = \sin t$; $z = t$. Find $\frac{dw}{dt}$

5)

Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ if $w = \log(x^2 + y^2)$ where $x = r \cos \theta, y = r \sin \theta$

6)

If $V = ze^{ax + by}$ and z is a homogenous function of degree n in x and y prove that $x \frac{\partial V}{\partial x} + y \frac{\partial V}{\partial y} = (ax + by + n)V$.

7)

Example 6.22 : Using Euler's theorem, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$ if

$$u = \sin^{-1} \left(\frac{x-y}{\sqrt{x} + \sqrt{y}} \right)$$

8)

Using Euler's theorem prove the following :

(i) If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x-y} \right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$.

9)

Verify Euler's theorem for $f(x,y) = \frac{1}{\sqrt{x^2 + y^2}}$

10)

Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ for the following functions :

$$u = \tan^{-1} \left(\frac{x}{y} \right)$$

11)

Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ for the following functions :

$$u = \frac{x}{y^2} - \frac{y}{x^2}$$

12)

Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ for the following functions :

$$u = \sin 3x \cos 4y$$

13)

Use differentials to find an approximate value for the given number

$$y = \sqrt[3]{1.02} + \sqrt[4]{1.02}$$

14)

If $w = u^2 e^v$ where $u = \frac{x}{y}$ and $v = y \log x$, find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$