**8) Find the axis, vertex, focus, directrix, equation of the latus rectum, length of the latus rectum for the following parabola and hence draw their graphs**

**$y^2$+8x-6y+1=0**

**9)**

Prove that the line $5x + 12y = 9$ touches the hyperbola $x^2 - 9y^2 = 9$ and find its point of contact.

**10 )**

Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Find the co-ordinates of the point of contact.

**11)**

Find the equation of the rectangular hyperbola which has for one of its asymptotes the line $x + 2y - 5 = 0$ and passes through the points (6, 0) and (– 3, 0).

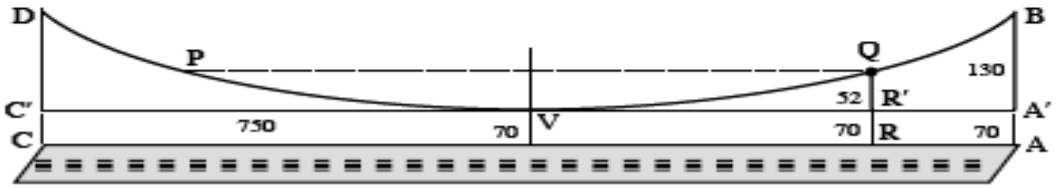
**12) find the equationof the hyperbola if its asymptotes are parallel to x + 2y -12 = 0 And x - 2y +8=0,(2,4) is the centre of the hyperbola and its passes through (2,0)**

**13)**

***Example 4.14 :***

A cable of a suspension bridge hangs in the form of a parabola when the load is uniformly distributed horizontally. The distance between two towers is 1500 ft, the points of support of the cable on the towers are 200ft above the road way and the lowest point on the cable is 70ft above the roadway. Find the vertical distance to the cable (parallel to the roadway) from a pole whose height is 122 ft.

**Solution :**




14 )

(5) A cable of a suspension bridge is in the form of a parabola whose span is 40 mts. The road way is 5 mts below the lowest point of the cable. If an extra support is provided across the cable 30 mts above the ground level, find the length of the support if the height of the pillars are 55 mts.

**15)**

The girder of a railway bridge is in the parabolic form with span 100 ft. and the highest point on the arch is 10 ft. above the bridge. Find the height of the bridge at 10 ft. to the left or right from the midpoint of the bridge.

**16)**

On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4mts when it is 6 mts away from the point of projection. Finally it reaches the ground 12 mts away from the starting point. Find the angle of projection

**17)**

Assume that water issuing from the end of a horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

**18 )**

A comet is moving in a parabolic orbit around the sun which is at the focus of a parabola. When the comet is 80 million kms from the sun, the line segment from the sun to the comet makes an angle of $\frac{\pi}{3}$ radians with the axis of the orbit find (i) the equation of the comet's orbit (ii) how close does the comet come nearer to the sun? (Take the orbit as open rightward).

**19 )**

*Example 4.32 :* An arch is in the form of a semi-ellipse whose span is 48 feet wide. The height of the arch is 20 feet. How wide is the arch at a height of 10 feet above the base?

**20 )**

The arch of a bridge is in the shape of a semi-ellipse having a horizontal span of 40ft and 16ft high at the centre. How high is the arch, 9ft from the right or left of the centre.

**21)**

*Example 4.33 :* The ceiling in a hallway 20ft wide is in the shape of a semi ellipse and 18 ft high at the centre. Find the height of the ceiling 4 feet from either wall if the height of the side walls is 12ft.

**22 )**

A satellite is travelling around the earth in an elliptical orbit having the earth at a focus and of eccentricity 1/2 . The shortest distance that the satellite gets to the earth is 400 kms. Find the longest distance that the satellite gets from the earth.

**23)**

The orbit of the planet mercury around the sun is in elliptical shape with sun at a focus. The semi-major axis is of length 36 million miles and the eccentricity of the orbit is 0.206. Find (i) how close the mercury gets to sun? (ii) the greatest possible distance between mercury and sun.

**24 )**

A kho-kho player in a practice session while running realises that the sum of the distances from the two kho-kho poles from him is always 8m. Find the equation of the path traced by him if the distance between the poles is 6m.

**25)**

*Example 4.35 :* A ladder of length 15m moves with its ends always touching the vertical wall and the horizontal floor. Determine the equation of the locus of a point $P$ on the ladder, which is 6m from the end of the ladder in contact with the floor.

## DIFFERENTIAL CALCULUS – APPLICATION – 1

| Qn.No | QUESTION | ANS |
|---|---|---|
| 1 | **The gradient of the curve $y = -2x + 3x + 5$ at $x = 2$ is** | **- 21** |
| 2 | **The rate of change of area A of a circle of radius *r* is** | $2\,n\,r\,d$ |
| 3 | **The velocity *v* of a particle moving along a straight line when at a distance *x* from the origin is given by $a + bv^2 = x^2$ where *a* and *b* are constants. Then the acceleration is** | **x/ b** |
| 4 | **A spherical snowball is melting in such a way that its volume is decreasing at a rate of 1 $cm^3$ / min. The rate at which the diameter is decreasing when the diameter is 10 cms is** | $\frac{1}{50\pi}$ cm / min |
| 5 | The slope of the tangent to the curve $y = 3x^2 + 3\sin x$ at $x = 0$ is | 3 |
| 6 | The slope of the normal to the curve $y = 3x^2$ at the point $x$ coordinate is 2 is | $\frac{-1}{12}$ |
| 7 | The point on the curve $y = 2x^2 - 6x - 4$ at which the tangent is to the $x-$ axis is | $\left(\frac{3}{2}, \frac{-17}{2}\right)$ |
| 8 | The equation of the tangent to the curve $y = \frac{x^3}{5}$ at the point $(-1, -1/5)$ is | $5y - 3x = 2$ |
| 9 | The equation of the normal to the curve $\theta = \frac{1}{t}$ at the point $(-3, -1/3)$ is | $3\theta = 27t + 80$ |
| 10 | The angle between the curves $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and $\frac{x^2}{8} - \frac{y^2}{8} = 1$ is | $\frac{\pi}{2}$ |
| 11 | The angle between the curve $y = e^{mx}$ and $y = e^{-mx}$ for $m > 1$ is | $\tan^{-1}\left(\frac{2m}{m^2-1}\right)$ |
| 12 | The parametric equations of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ are | $x = a\cos^3\theta; y = a\sin^3\theta$ |
| 13 | If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle $\theta$ with the $x-$ axis then the slope of the normal is | $\tan\theta$ |
| 14 | If the length of the diagonal of a square is increasing at the rate of 0.1 cm / sec. What is the rate of increase of its area when the side is $\frac{15}{\sqrt{2}}$ cm? | 1.5 $cm^2$/sec |
| 15 | What is the surface area of a sphere when the volume is increasing at the same rate as its radius? | 1 |

| | | |
|---|---|---|
| **16** | For what values of $x$ is the rate of increase of $x^3 - 2x^2 + 3x + 8$ is twice the rate of increase of $x$ | $\left(\frac{1}{3}, 3\right)$ |
| **17** | The radius of a cylinder is increasing at the rate of 2cm / sec and its altitude is decreasing at the rate of 3cm / sec. The rate of change of volume when the radius is 3cm and the altitude is 5cm is | $33\pi$ |
| **18** | If $y = 6x - x^3$ and $x$ increases at the rate of 5 units per second, the rate of change of slope when $x = 3$ is | – 90 units / sec( |
| **19** | If the volume of an expanding cube is increasing at the rate of $4\text{cm}^3$ / sec then the rate of change of surface area when the volume of the cube is 8 cubic cm is | $8\text{cm}^2/\text{sec}$ |
| **20** | The gradient of the tangent to the curve $y = 8 + 4x - 2x^2$ at the point where the curve cuts the $y$-axis is | 4 |
| **21** | The Angle between the parabolas $y^2 = x$ and $x^2 = y$ at the origin is | $\frac{\pi}{2}$ |
| **22** | For the curve $x = e^t \cos t$ ; $y = e^t \sin t$ the tangent line is parallel to the $x$-axis when $t$ is equal to | $-\frac{\pi}{4}$ |
| **23** | If a normal makes an angle $\theta$ with positive $x$-axis then the slope of the curve at the point where the normal is drawn is | $-\cot\theta$ |
| **24** | The value of '$a$' so that the curves $y = 3e^x$ and $y = \frac{a}{3} e^{-x}$ intersect orthogonally is | 1 |
| **25** | If $s = t^3 - 4t^2 + 7$, the velocity when the acceleration is zero is | $\frac{-16}{3}$ m/sec |
| **26** | If the velocity of a particle moving along a straight line is directly proportional to the square of its distance from a fixed point on the line. Then its acceleration is proportional to | $s^3$ |
| **27** | The Rolle's constant for the function $y = x^2$ on $[-2, 2]$ is | 0 |
| **28** | The '$c$' of Lagranges Mean Value Theorem for the function $f(x) = x^2 + 2x - 1$ ; $a = 0$, $b = 1$ is | $\frac{1}{2}$ |
| **29** | The value of $c$ in Rolle's Theorem for the function $f(x) = \cos\frac{x}{2}$ on $[\pi, 3\pi]$ is | $2\pi$ |
| **30** | The value of '$c$' of Lagranges Mean Value Theorem for $f(x) = \sqrt{x}$ when $a = 1$ and $b = 4$ is | $\frac{9}{4}$ |
| **31** | $\lim_{x \to \infty} \frac{x^2}{e^x}$ is | 0 |
| **32** | $\lim_{x \to 0} \frac{a^x - b^x}{c^x - d^x}$ | $\frac{\log(a/b)}{\log(c/d)}$ |

| | | |
|---|---|---|
| **33** | If $f(a) = 2;\ f'(a) = 1\ ;\ g(a) = -1\ ;\ g'(a) = 2$ then the value of $\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$ is | 5 |
| **34** | Which of the following function is increasing in $(0, \infty)$ | $e^x$ |
| **35** | The function $f(x) = x^2 - 5x + 4$ is increasing in | $(4, \infty)$ |
| **36** | The function $f(x) = x^2$ is decreasing in | $(-\infty, 0)$ |
| **37** | The function $y = \tan x - x$ is | **An increasing** $(0, \frac{\Pi}{2})$ |
| **38** | In a given semi circle of diameter 4 cm a rectangle is to be inscribe The maximum area of the rectangle is | 4 |
| **39** | The least possible perimeter of a rectangle of area $100m^2$ is | 40 |
| **40** | If $f(x) = x^2 - 4x + 5$ on $[0, 3]$ then the absolute maximum value is | 5 |
| **41** | The curve $y = -e^{-x}$ is | **Everywhere concave downward** |
| **42** | Which of the following curves is concave down? | $y = -x^2$ |
| **43** | The point of inflexion of the curve $y = x^4$ is at | nowhere |
| **44** | The curve $y = ax^3 + bx^2 + cx + d$ has a point of inflexion at $x = 1$ then | $3a + b = 1$ |