COMPLEX NUMBERS
PART-AQUESTIONS

1.If $-1+3$ is a root of $x^2 - 6x + k = 0$ then the value of k is

2. If co is the cube root of unity then the value of
$(1 - co)(1 - co^2)(1 - co^4)(1 - co^8)$ is

3. If $a = 3 + i$ and $z = 2 - 3i$ then the points on the Argand diagram representing az, $3az$ and $-az$ are

4. If $x^2 + y^2 = 1$ then the value of $\frac{1+x+iy}{1+x-iy}$ is

5. If the amplitude of a complex number is $\pi/2$ then the number is

6. If $a = \cos\alpha - i\sin\alpha$, $b = \cos\beta - i\sin\beta$
$c = \cos\gamma - i\sin\gamma$ then $(a^2c^2 - b^2)/abc$ is

7. The modulus and amplitude of the complex number $\left[e^{3-i\pi/4}\right]^3$ are respectively

8 If $z_n = \cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}$ then $z_1 z_2 \ldots z_6$ is

9. The modulus of the complex number $2 + i\sqrt{3}$ is

10 If $-i + 2$ is one root of the equation $ax^2 - bx + c = 0$, then the other root is

11 The equation having $4 - 3i$ and $4 + 3i$ as roots is

12 The conjugate of $i^{13} + i^{14} + i^{15} + i^{16}$ is

13. If $\frac{1-i}{1+i}$ is a root of the equation $ax^2 + bx + 1 = 0$, where a, b are real then (a, b) is

14. If $-\overline{z}$ lies in the third quadrant then z lies in the

15 The quadratic equation whose roots are $\pm i\sqrt{7}$ is

16 If ω is a cube root of unity then the value of $(1 - \omega + \omega^2)^4 + (1 + \omega - \omega^2)^4$ is

17 The value of $i + i^{22} + i^{23} + i^{24} + i^{25}$ is

18 The value of $\left[\frac{-1+i\sqrt{3}}{2}\right]^{100}+\left[\frac{-1-i\sqrt{3}}{2}\right]^{100}$ is

19 If $(m-5)+i(n+4)$ is the complex conjugate of $(2m+3)+i(3n-2)$ then (n, m) are

20 The polar form of the complex number $(i^{25})^3$ is

21 $\frac{1+e^{-i\theta}}{1+e^{i\theta}} =$

22 If P represents the variable complex number z and if $|2z-1| = 2|z|$ then the locus of P is

23 The points z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallelogram taken in order if and only if

24 If z represents a complex number then arg (z) + arg $(\overline{z})$ is

25 If $x = \cos\theta + i\sin\theta$ the value of $x^n + \frac{1}{x^n}$ is

26 $z_1 = 4+5i,\ z_2 = -3+2i$ then $\frac{z_1}{z_2}$ is

27 If the point represented by the complex number iz is rotated about the origin through the angle $\frac{\pi}{2}$ in the counter clockwise direction then the complex number representing the new position is

28 If ω is the nth root of unity then

29 If $A+iB = (a_1+ib_1)(a_2+ib_2)(a_3+ib_3)$ then A^2+B^2 is

PART-B(6MARKQUESTIONS)

1. Find the square root of (- 7 + 24 i).

2. Find the square root of (- 8 - 6i)

3. Prove that the triangle formed by the points representing the complex numbers (10 + 8i), (- 2 + 4i) and (- 11 + 31i) on the Argand plane is right angled.

TRY YOURSELF:

4. *Example 3.13 :* Prove that the complex numbers $3 + 3i, -3 - 3i, -3\sqrt{3} + 3\sqrt{3}\,i$ are the vertices of an equilateral triangle in the complex plane.

5. *Example 3.14 :* Prove that the points representing the complex numbers $2i$, $1 + i$, $4 + 4i$ and $3 + 5i$ on the Argand plane are the vertices of a rectangle.

6. *Example 3.15 :* Show that the points representing the complex numbers $7 + 9i, -3 + 7i, 3 + 3i$ form a right angled triangle on the Argand diagram.

7. Prove that the points representing the complex numbers $(7 + 5i)$, $(5 + 2i)$, $(4 + 7i)$ and $(2 + 4i)$ form a parallelogram. (Plot the points and use midpoint formula).

8. P represents the variable complex number z. Find the locus of P, if $|z - 5i| = |z + 5i|$

TRY YOURSELF:

* P represents the variable complex number z. Find the locus of P, if

* P represents the variable complex number z. Find the locus of P, if $|2z - 3| = 2$

* Solve the equation $x^4 - 8x^3 + 24x^2 - 32x + 20 = 0$ if $3 + i$ is a root.

TRY YOURSELF:

* Solve the equation $x^4 - 4x^3 + 11x^2 - 14x + 10 = 0$ if one root is $1 + 2i$

* Solve : $6x^4 - 25x^3 + 32x^2 + 3x - 10 = 0$ given that one of the roots is $2 - i$

* *Example 3.17 :* Solve the equation $x^4 - 4x^2 + 8x + 35 = 0$, if one of its roots is $2 + \sqrt{3}\,i$

9. Prove that

(i) $(1 + i)^n + (1 - i)^n = 2^{\frac{n+2}{2}} \cos\frac{n\pi}{4}$

TRY YOURSELF:

* $(\sqrt{3} + i)^n + (\sqrt{3} - i)^n = 2^{n+1} \cos\frac{n\pi}{6}$

* $(1+i\sqrt{3})^n + (1-i\sqrt{3})^n = 2^{n+1}\cos\frac{n\pi}{3}$

7.Prove that $(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n(\theta/2)\cos\frac{n\theta}{2}$

8. Simplify : $\frac{(\cos\alpha + i\sin\alpha)^3}{(\sin\beta + i\cos\beta)^4}$

9. Simplify : $\frac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5}$

10 If $x + \frac{1}{x} = 2\cos\theta$ prove that

(i) $x^n + \frac{1}{x^n} = 2\cos n\theta$ (ii) $x^n - \frac{1}{x^n} = 2i\sin n\theta$

11 If $x + \frac{1}{x} = 2\cos\theta$ and $y + \frac{1}{y} = 2\cos\phi$ show that

(i) $\frac{x^m}{y^n} + \frac{y^n}{x^m} = 2\cos(m\theta - n\phi)$ (ii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\theta - n\phi)$

12 If $x = \cos\alpha + i\sin\alpha$; $y = \cos\beta + i\sin\beta$

prove that $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

13. For any two complex numbers z_1 and z_2

(i) $|z_1 z_2| = |z_1| . |z_2|$ (ii) $\arg(z_1 . z_2) = \arg z_1 + \arg z_2$ (oct-2007,june-2008)

14 (iii) Triangle inequality :

The modules of sum of two complex numbers is always less than or equal to the sum of their moduli.

i.e., $|z_1 + z_2| \leq |z_1| + |z_2|$

15Solve: $x^4 + 4 = 0$

Analyticalgeometry

Part-A

S.No	question	
1	The axis of the parabola $y^2 - 2y + 8x - 23 = 0$ is	
2	The line $4x + 2y = c$ is a tangent to the parabola $y^2 = 16x$ then c is	
3	The line $4x + 2y = c$ is a tangent to the parabola $y^2 = 16x$ then c is	
4	The point of intersection of the tangents at $t_1 = t$ and $t_2 = 3t$ to the parabola $y^2 = 8x$ is	
5	The length of the latus rectum of the parabola $y^2 - 4x + 4y + 8 = 0$ is	
6	The diretrix of the parabola $y^2 = x + 4$ is	
7	The length of the latus rectum of the parabola whose vertex is $(2, -3)$ and the directrix $x = 4$ is	
8	The focus of the parabola $x^2 = 16y$ is	
9	The vertex of the parabola $x^2 = 8y - 1$ is	
10	The line $2x + 3y + 9 = 0$ touches the parabola $y^2 = 8x$ at the point	
11	The tangents at the end of any focal chord to the parabola $y^2 = 12x$ intersect on the line	
12	The angle between the two tangents drawn from the point $(-4, 4)$ to $y^2 = 16x$ is	
13	The eccentricity of the conic $9x^2 + 5y^2 - 54x - 40y + 116 = 0$ is	
14	The length of the semi-major and the length of semi minor axis of the ellipse $\frac{x^2}{144} + \frac{y^2}{169} = 1$ are	
15	The distance between the foci of the ellipse $9x^2 + 5y^2 = 180$ is	
16	If the length of major and semi-minor axes of an ellipse are 8, 2 and their corresponding equations are $y - 6 = 0$ and $x + 4 = 0$ then the equations of the ellipse is	
17	The straight line $2x - y + c = 0$ is a tangent to the ellipse $4x^2 + 8y^2 = 32$ if c is	
18	The sum of the distance of any point on the ellipse $4x^2 + 9y^2 = 36$ from $(\sqrt{5}, 0)$and $(-\sqrt{5}, 0)$is	
19	The radius of the director circle of the conic $9x^2 + 16y^2 = 144$ is	
20	The locus of foot of perpendicular from the focus to a tangent of the curve $16x^2 + 25y^2 = 400$ is	
21	The eccentricity of the hyperbola $12y^2 - 4x^2 - 24x + 48y - 127 = 0$ is	
22	The eccentricity of the hyperbola whose latus rectum is equal to half of its conjugate axis is	
23	The difference between the focal distances of any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is 24 and the eccentricity is 2. Then the equation of the hyperbola is	
24	The directrices of the hyperbola $x^2 - 4(y - 3)^2 = 16$ are	
25	The line $5x - 2y + 4k = 0$ is a tangent to $4x^2 - y^2 = 36$ then k is	

26	The equation of the chord of contact of tangents from (2, 1) to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ is	
27	The angle between the asymptotes to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ is	
28	The asymptotes of the hyperbola $36y^2-25x^2+900=0$ are	
29	The product of the perpendiculars drawn from the point (8, 0) on the hyperbola to its asymptotes is $\frac{x^2}{64}-\frac{y^2}{36}=1$ is	
30	The locus of the point of intersection of perpendicular tangents to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ is	
31	The eccentricity of the hyperbola with asymptotes $x + 2y - 5 = 0$, $2x - y + 5 = 0$ is	
32	Length of the semi-transverse axis of the rectangular hyperbola $xy = 8$ is	
33	The asymptotes of the rectangular hyperbola $xy = c^2$ are	
34	The co-ordinate of the vertices of the rectangular hyperbola $xy = 16$ are	
35	One of the foci of the rectangular hyperbola $xy = 18$ is	
36	The length of the latus rectum of the rectangular hyperbola $xy = 32$ is	
37	The area of the triangle formed by the tangent at any point on the rectangular hyperbola $xy = 72$ and its asymptotes is	
38	The normal to the rectangular hyperbola $xy = 9$ at $\left(6, \frac{3}{2}\right)$ meets the curve again at	

Part-C

ELLIPSE

1. Find the eccentricity, centre, foci, vertices of the following ellipses and draw the diagram :

$x^2 + 4y^2 - 8x - 16y - 68 = 0$

2. Find the eccentricity, centre, foci, vertices of the following ellipses and draw the diagram :

$36x^2 + 4y^2 - 72x + 32y - 44 = 0$

TRY YOURSELF

* Find the eccentricity, centre, foci, vertices of the following ellipses and draw the diagram :

$16x^2 + 9y^2 + 32x - 36y = 92$

HYPERBOLA

3. Find the eccentricity, centre, foci and vertices of the following hyperbolas and draw their diagrams.

$x^2 - 3y^2 + 6x + 6y + 18 = 0$

TRY YOURSELF

- Find the eccentricity, centre, foci and vertices of the following hyperbolas and draw their diagrams.

$9x^2 - 16y^2 + 36x + 32y + 164 = 0$

4. Find the eccentricity, centre, foci and vertices of the following hyperbolas and draw their diagrams.

$x^2 - 4y^2 + 6x + 16y - 11 = 0$

TRY YOURSELF

*

Example 4.56 : Find the eccentricity, centre, foci and vertices of the hyperbola $9x^2 - 16y^2 - 18x - 64y - 199 = 0$ and also trace the curve.

5. Find the axis, vertex, focus, equation of directrix, latus rectum, length of the latus rectum for the following parabolas and hence sketch their graphs.

$x^2 - 6x - 12y - 3 = 0$

Example 4.7 : Find the axis, vertex, focus, directrix, equation of the latus rectum, length of the latus rectum for the following parabolas and hence draw

6. their graphs.

$x^2 - 2x + 8y + 17 = 0$

Example 4.7 : Find the axis, vertex, focus, directrix, equation of the latus rectum, length of the latus rectum for the following parabolas and hence draw

7. their graphs.

$y^2 - 8x + 6y + 9 = 0$