APPLICATIONS OF MATRICES AND DETERMINANTS

PART – A (Questions and Answers)

1. If the rank of the matrix A is 3 then det(KA) = _____
2. If the rank of the matrix $\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{bmatrix}$ is 2, then λis _____
3. If I is the unit matrix of order n, where $k \neq 0$ is a constant, then adj(KI) = _____
4. If A = [2 0 1], then rank of $A A^T$ is _____
5. If the matrix $\begin{bmatrix} -1 & 3 & 2 \\ 1 & k & -3 \\ 1 & 4 & 5 \end{bmatrix}$ has an inverse then the values of k = _____
6. If A and B are any two matrices such that AB = 0 and A is non-singular, then _____
7. The system of equations ax + y + z =0 ; x + by +z = 0; x + y +cz = 0 has a non-trivial solution then $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=$ _____
8. If A = $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then the rank of AA^T is _____
9. The rank of the diagonal matrix $\begin{bmatrix} 1 & & & \\ & 2 & & \\ & & 0 & \\ & & & -4 \\ & & & & 0 \end{bmatrix}$
10. If $A = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$, then A^{12} is
11. If A is a scalar matrix with scalar $k \neq 0$, of order 3, then A^{-1} is
12. Inverse of $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ is
13. The inverse of the matrix $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ is
14. In a system of 3 linear non-homogeneous equation with three unknowns, if $\Delta = 0$ and $\Delta_x = 0$, $\Delta_y \neq 0$ and $\Delta_z = 0$ then the system has
15. If $ae^x + be^y = c$; $pe^x + qe^y = d$ and $\Delta_1 = \begin{vmatrix} a & b \\ p & q \end{vmatrix}$; $\Delta_2 = \begin{vmatrix} c & b \\ d & q \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & c \\ p & d \end{vmatrix}$ then the value of (x, y) is

16. The rank of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 4 & -4 & 8 \end{bmatrix}$ is

17. If the equation $\begin{aligned} -2x + y + z &= l \\ x - 2y + z &= m \\ x + y - 2z &= n \end{aligned}$ such that $l + m + n = 0$, then the system has

18. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, then (adj A) A =

19. If A is a square matrix of order n then | adj A | is

PART – B

Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and verify the result

1. A (adj A) = (adj A)A = | A | . I

TRY YOURSELF

Find the adjoint of the matrix A = $\begin{bmatrix} -1 & 2 \\ 1 & -4 \end{bmatrix}$ and verify the result A(adjA) = (adjA) = IAI,I

If $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ verify that

2. (i) $(AB)^{-1} = B^{-1} A^{-1}$ (ii) $(AB)^T = B^T A^T$

Try Yourself

If A = $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$ verify that (i) $(AB)^{-1} = B^{-1}A^{-1}$ (ii) $(AB)^T = \mathbf{B}^T A^T$

3. Solve by matrix inversion method of the system of linear equations 2x – y = 7, 3x -2y = 11

Try Yourself

Solve by matrix inversion method each of the following system of linear equations:

(i) 7x + 3y = -1, 2x + y = 0

(ii) x + y = 3, 2x + 3y = 8

4. Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

5. For A = $\begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{bmatrix}$, Show that A = A^{-1}

6. Show that the adjoint of A = $\begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ is A itself

Try Yourself

Find the inverse of each of the following matrices

1. $\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ 2. $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ 3. $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ 4. $\begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & 4 \end{bmatrix}$

5. $\begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

7. Find the rank of the matrix $\begin{bmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{bmatrix}$

8. Find the rank of the matrix $\begin{bmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{bmatrix}$

9. Find the rank of the matrix $\frac{1}{4}\begin{bmatrix} -4 & 12 & 12 & 4 \\ 0 & 4 & 8 & 4 \\ 4 & -4 & 8 & 0 \end{bmatrix}$

Try Yourself:

Find the rank of the following matrices

1. $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & -3 & 0 & -1 \\ 1 & 1 & -1 & 0 \end{bmatrix}$ 2. $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$ 3. $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{bmatrix}$

4. $\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$ 5. $\begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & 4 & 6 & -2 \\ 3 & 6 & 9 & -3 \end{bmatrix}$

10. Solve the non-homogeneous system of linear equations by determinant method:
$4x + 5y = 9,\ 8x + 10y = 18.$

Try Yourself:

Solve the following non-homogeneous system of linear equations by determinant method:

(i) $2x + 3y = 8\ ,\ 4x + 6y = 16$

(ii) $2x - 3y = 7\ ,\ 4x - 6y = 14$

11. Solve the following non-homogeneous equations of three unknowns.
$2x + 2y + z = 5,\ x - y + z = 1,\ 3x + y + 2z = 4.$

12. Solve the following non-homogeneous equations of three unknowns.
$x + y + 2z = 4,\ 2x + 2y + 4z = 8,\ 3x + 3y + 6z = 10.$

13. Examine the consistency of the following system of equations. If it is consistent then solve the same.

(i) $x + y + z = 7,\ x + 2y + 3z = 18,\ y + 2z = 6.$

(ii) $x - 4y + 7z = 14,\ 3x + 8y - 2z = 13,\ 7x - 8y + 26z = 5.$

14.Solve: $x + y + 2z = 0$, $3x + 2y + z = 0$, $2x + y – z = 0$.

15.State and Prove Reversal Law for Inverses.

PART – C

1. Solve by matrix inversion method each of the following system of linear equations
   (i) $x – 3y – 8z + 10 = 0$, $3x + y = 4$, $2x + 5y + 6z = 13$
   (ii) $2x – y + 3z = 9$, $x + y + z = 6$, $x – y + z = 2$
   (iii) $X + y + z = 9$, $2x + 5y + 7z = 52$, $2x + y – z = 0$
   (iv) $2x – y + z = 7$, $3x + y – 5z = 13$, $x + y + z = 5$.
2. Solve the following system of linear equations by determinant method.
   (i) $\frac{1}{x}+\frac{1}{y}-\frac{1}{z}=1,\ \frac{2}{x}+\frac{4}{y}+\frac{1}{z}=5,\ \frac{3}{x}-\frac{2}{y}-\frac{2}{z}=0$
   (ii) $x + 2y+ z= 7$, $2x – y + 2z = 4$, $x + y + - 2z = -1$
   (iii) $2x + y + z = 5$, $x + y + z =4$, $x – y + 2z = 1$
   (iv) $x + y + z =4$, $x – y + z = 2$, $2x + y – z = 1$
   (v) $3x + y – z = 2$, $2x – y + 2z = 6$, $2x + y – 2z = -2$
3. Solve the following non-homogeneous equations of three unknowns
   (i) $x + 2y + z = 6$, $3x + 3y – z = 3$, $2x + y – 2z = -3$
   (ii) $2x + y – z = 4$, $x + y – 2z = 0$, $3x + 2y – 3z = 4$
   (iii) $X + y + 2z = 6$, $3x + y – z = 2$, $4x + 2y + z = 8$
   (iv) $X + 2y + 3z = 6$, $x + y + z = 3$, $2x + 3y + 4z = 9$

Try Yourself:

1. A bag contains 3 types of coins namely Re.1, Rs.2, Rs.5. There are 30 coins amounting to Rs.100 in total. Find the number of coins in each category.
2. A small seminar hall can hold 100 chairs. Three different colours (red, blue and green) of chairs are available. The cost of a red chair is Rs.240, cost of a blue chair is Rs.260 and the cost of a green chair is Rs.300. The total cost of chair is Rs. 25,000. Find atleast 3 different solution of the number of chairs in each colour to be purchased.

4. Solve the following non-homogeneous system of linear equations by determinant method:
   (i) $2x – y + z = 2$, $6x – 3y + 3z = 6$, $4x – 2y + 2z = 4$
   (ii) $x + y + 2z = 4$, $2x + 2y + 4z = 8$, $3x + 3y + 6z = 12$
   (iii) $x + 2y + 3z = 6$, $2x + 4y + 6z = 12$, $3x + 6y + 9z = 18$
5. Examine the consistency of the following system of equations. If it is consistent then solve the same.
   (i) $4x + 3y + 6z = 25$, $x + 5y + 7z = 13$, $2x + 9y + z = 1$
   (ii) $2x + 5y + 7z = 52$, $x + y + z = 9$, $2x + y – z = 0$
6. Examine the consistency of the following system of equations. If it is consistent then solve the same.
   (i) $x – 3y – 8z = -10$, $3x + y – 4z = 0$, $2x + 5y + 6z – 13 = 0$
   (ii) $x + y + z = 6$, $x + 2y + 3z = 14$, $x + 4y + 7z = 30$
7. Examine the consistency of the following system of equations. If it is consistent then solve the same.
   (i) $x + y – z = 1$, $2x + 2y – 2z = 2$, $-3x -3y + 3z = -3$
   (ii) $x – y + z = 5$, $-x + y – z = -5$, $2x – 2y + 2z = 10$

8. Examine the consistency of the equations $2x - 3y + 7z = 5$, $3x + y - 3z = 13$, $2x + 19y - 47z = 32$.
9. Discuss the solutions of the system of equations for all values of λ, $X + y + z = 2$, $2x + y - 2z = 2$, $\lambda x + y + 4z = 2$.
10. For what values of k , the system of equations $Kx + y + z = 1$, $x + ky + z = 1$, $x + y + kz = 1$ have (i) unique solution (ii) more than one solution (iii) no solution.

Try Yourself:

(*) Investigate for what values of λ, μ the simultaneous equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.

(**) For what value of μ the equations $x + y + 3z = 0$, $4x + 3y + \mu z = 0$, $2x + y + 2z = 0$ have a (i) trivial solution, (ii) non-trivial solution.